1 **a**

$$\begin{array}{r} x^2 + x - 2 \\ x+1 \overline{)\, x^3 + 2x^2 - x - 2} \\ \underline{x^3 + x^2} \\ x^2 - x \\ \underline{x^2 + x} \\ -2x - 2 \\ \underline{-2x - 2} \end{array}$$

quotient: $x^2 + x - 2$

b

$$\begin{array}{r} x^2 + 4x - 1 \\ x-2 \overline{)\, x^3 + 2x^2 - 9x + 2} \\ \underline{x^3 - 2x^2} \\ 4x^2 - 9x \\ \underline{4x^2 - 8x} \\ -x + 2 \\ \underline{-x + 2} \end{array}$$

quotient: $x^2 + 4x - 1$

c

$$\begin{array}{r} x^2 - x + 5 \\ x+4 \overline{)\, x^3 + 3x^2 + x + 20} \\ \underline{x^3 + 4x^2} \\ -x^2 + x \\ \underline{-x^2 - 4x} \\ 5x + 20 \\ \underline{5x + 20} \end{array}$$

quotient: $x^2 - x + 5$

d

$$\begin{array}{r} 2x^2 + x - 3 \\ x-1 \overline{)\, 2x^3 - x^2 - 4x + 3} \\ \underline{2x^3 - 2x^2} \\ x^2 - 4x \\ \underline{x^2 - x} \\ -3x + 3 \\ \underline{-3x + 3} \end{array}$$

quotient: $2x^2 + x - 3$

e

$$\begin{array}{r} 6x^2 + 11x - 18 \\ x-5 \overline{)\, 6x^3 - 19x^2 - 73x + 90} \\ \underline{6x^3 - 30x^2} \\ 11x^2 - 73x \\ \underline{11x^2 - 55x} \\ -18x + 90 \\ \underline{-18x + 90} \end{array}$$

quotient: $6x^2 + 11x - 18$

f

$$\begin{array}{r} -x^2 + 7x - 4 \\ x+2 \overline{)\, -x^3 + 5x^2 + 10x - 8} \\ \underline{-x^3 - 2x^2} \\ 7x^2 + 10x \\ \underline{7x^2 + 14x} \\ -4x - 8 \\ \underline{-4x - 8} \end{array}$$

quotient: $-x^2 + 7x - 4$

g

$$\begin{array}{r} x^2 - 3x + 7 \\ x+3 \overline{)\, x^3 + 0x^2 - 2x + 21} \\ \underline{x^3 + 3x^2} \\ -3x^2 - 2x \\ \underline{-3x^2 - 9x} \\ 7x + 21 \\ \underline{7x + 21} \end{array}$$

quotient: $x^2 - 3x + 7$

h

$$\begin{array}{r} 3x^2 - 2x + 12 \\ x+6 \overline{)\, 3x^3 + 16x^2 + 0x + 72} \\ \underline{3x^3 + 18x^2} \\ -2x^2 + 0x \\ \underline{-2x^2 - 12x} \\ 12x + 72 \\ \underline{12x + 72} \end{array}$$

quotient: $3x^2 - 2x + 12$

2 **a**

$$\begin{array}{r} x^2 + 3x + 2 \\ x+5 \overline{)\, x^3 + 8x^2 + 17x + 16} \\ \underline{x^3 + 5x^2} \\ 3x^2 + 17x \\ \underline{3x^2 + 15x} \\ 2x + 16 \\ \underline{2x + 10} \\ 6 \end{array}$$

quotient: $x^2 + 3x + 2$ remainder: 6

b

$$\begin{array}{r} x^2 - 8x + 5 \\ x-7 \overline{)\, x^3 - 15x^2 + 61x - 48} \\ \underline{x^3 - 7x^2} \\ -8x^2 + 61x \\ \underline{-8x^2 + 56x} \\ 5x - 48 \\ \underline{5x - 35} \\ -13 \end{array}$$

quotient: $x^2 - 8x + 5$ remainder: -13

c

$$\begin{array}{r} 3x^2 - 2x + 4 \\ x+2 \overline{)\, 3x^3 + 4x^2 + 0x + 7} \\ \underline{3x^3 + 6x^2} \\ -2x^2 + 0x \\ \underline{-2x^2 - 4x} \\ 4x + 7 \\ \underline{4x + 8} \\ -1 \end{array}$$

quotient: $3x^2 - 2x + 4$ remainder: -1

d

$$\begin{array}{r} -x^2 + 3x - 9 \\ x+8 \overline{)\, -x^3 - 5x^2 + 15x - 50} \\ \underline{-x^3 - 8x^2} \\ 3x^2 + 15x \\ \underline{3x^2 + 24x} \\ -9x - 50 \\ \underline{-9x - 72} \\ 22 \end{array}$$

quotient: $-x^2 + 3x - 9$ remainder: 22

e

$$\begin{array}{r} 4x^2 + 14x + 26 \\ x-3 \overline{)\, 4x^3 + 2x^2 - 16x + 3} \\ \underline{4x^3 - 12x^2} \\ 14x^2 - 16x \\ \underline{14x^2 - 42x} \\ 26x + 3 \\ \underline{26x - 78} \\ 81 \end{array}$$

quotient: $4x^2 + 14x + 26$ remainder: 81

f

$$\begin{array}{r} -6x^2 - 10x + 20 \\ x+2 \overline{)\, -6x^3 - 22x^2 + 0x + 1} \\ \underline{-6x^3 - 12x^2} \\ -10x^2 + 0x \\ \underline{-10x^2 - 20x} \\ 20x + 1 \\ \underline{20x + 40} \\ -39 \end{array}$$

quotient: $-6x^2 - 10x + 20$ remainder: -39

3 **a** let $f(x) \equiv x^3 + 2x^2 - 2x - 1$
$f(1) = 1 + 2 - 2 - 1 = 0$
$\therefore$ $(x - 1)$ is a factor

b let $f(x) \equiv x^3 - 5x^2 - 9x + 2$
$f(-2) = -8 - 20 + 18 + 2 = -8$
$\therefore$ $(x + 2)$ is not a factor

c let $f(x) \equiv x^3 - x^2 - 14x + 27$
$f(3) = 27 - 9 - 42 + 27 = 3$
$\therefore$ $(x - 3)$ is not a factor

d let $f(x) \equiv 2x^3 + 13x^2 + 2x - 24$
$f(-6) = -432 + 468 - 12 - 24 = 0$
$\therefore$ $(x + 6)$ is a factor

e let $f(x) \equiv 2x^3 - 5x^2 + 7x - 14$
$f(-\frac{1}{2}) = -\frac{1}{4} - \frac{5}{4} - \frac{7}{2} - 14 = -19$
$\therefore$ $(2x + 1)$ is not a factor

f let $f(x) \equiv 2 - 17x + 25x^2 - 6x^3$
$f(\frac{2}{3}) = 2 - \frac{34}{3} + \frac{100}{9} - \frac{16}{9} = 0$
$\therefore$ $(3x - 2)$ is a factor

4 **a** $f(1) = 1 - 2 - 11 + 12 = 0$
$\therefore (x - 1)$ is a factor of $f(x)$

b

$$\begin{array}{r} x^2 - x - 12 \\ x-1 \overline{)\, x^3 - 2x^2 - 11x + 12} \\ \underline{x^3 - x^2} \\ -x^2 - 11x \\ \underline{-x^2 + x} \\ -12x + 12 \\ \underline{-12x + 12} \end{array}$$

$\therefore f(x) \equiv (x - 1)(x^2 - x - 12)$
$\equiv (x - 1)(x + 3)(x - 4)$

5 $g(-3) = -54 + 9 + 39 + 6 = 0$
$\therefore (x + 3)$ is a factor of $g(x)$

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x+3 \overline{)\, 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 + 6x^2} \\ -5x^2 - 13x \\ \underline{-5x^2 - 15x} \\ 2x + 6 \\ \underline{2x + 6} \end{array}$$

$\therefore g(x) \equiv (x + 3)(2x^2 - 5x + 2)$
$\equiv (x + 3)(2x - 1)(x - 2)$

$g(x) = 0 \Rightarrow (x + 3)(2x - 1)(x - 2) = 0$
$x = -3, \frac{1}{2}$ or 2

6 $f(4) = 0 \quad \therefore (x - 4)$ is a factor of $f(x)$

$$\begin{array}{r} 6x^2 + 17x - 3 \\ x-4 \overline{)\, 6x^3 - 7x^2 - 71x + 12} \\ \underline{6x^3 - 24x^2} \\ 17x^2 - 71x \\ \underline{17x^2 - 68x} \\ -3x + 12 \\ \underline{-3x + 12} \end{array}$$

$\therefore f(x) \equiv (x - 4)(6x^2 + 17x - 3)$
$\equiv (x - 4)(6x - 1)(x + 3)$

$f(x) = 0 \Rightarrow (x - 4)(6x - 1)(x + 3) = 0$

$x = -3, \frac{1}{6}$ or 4

7 **a** $g(-2) = 0 \quad \therefore (x + 2)$ is a factor of $g(x)$

$$\begin{array}{r} x^2 + 5x - 3 \\ x+2 \overline{)\, x^3 + 7x^2 + 7x - 6} \\ \underline{x^3 + 2x^2} \\ 5x^2 + 7x \\ \underline{5x^2 + 10x} \\ -3x - 6 \\ \underline{-3x - 6} \end{array}$$

$\therefore g(x) \equiv (x + 2)(x^2 + 5x - 3)$

b other solutions given by $x^2 + 5x - 3 = 0$

$x = \frac{-5 \pm \sqrt{25 + 12}}{2} = \frac{-5 \pm \sqrt{37}}{2}$

$x = -5.54$ or 0.54

8 **a** $f(1) = 1 + 2 - 11 - 12 = -20$
$f(2) = 8 + 8 - 22 - 12 = -18$
$f(-1) = -1 + 2 + 11 - 12 = 0$
$f(-2) = -8 + 8 + 22 - 12 = 10$

b $(x + 1)$ is a factor of $f(x)$

$$\begin{array}{r} x^2 + x - 12 \\ x+1 \overline{)\, x^3 + 2x^2 - 11x - 12} \\ \underline{x^3 + x^2} \\ x^2 - 11x \\ \underline{x^2 + x} \\ -12x - 12 \\ \underline{-12x - 12} \end{array}$$

$\therefore f(x) = (x + 1)(x^2 + x - 12)$
$= (x + 1)(x + 4)(x - 3)$

9 **a** let $f(x) = x^3 - 2x^2 - 5x + 6$
$f(1) = 0$
$\therefore (x - 1)$ is a factor

$$\begin{array}{r|l} x-1 & x^2 - x - 6 \\ \hline & x^3 - 2x^2 - 5x + 6 \\ & x^3 - x^2 \\ \hline & -x^2 - 5x \\ & -x^2 + x \\ \hline & -6x + 6 \\ & -6x + 6 \\ \hline \end{array}$$

$\therefore$
$f(x) = (x - 1)(x^2 - x - 6)$
$= (x - 1)(x + 2)(x - 3)$

b let $f(x) = x^3 + x^2 - 5x - 2$
$f(1) = -5$, $f(2) = 0$
$\therefore (x - 2)$ is a factor

$$\begin{array}{r|l} x-2 & x^2 + 3x + 1 \\ \hline & x^3 + x^2 - 5x - 2 \\ & x^3 - 2x^2 \\ \hline & 3x^2 - 5x \\ & 3x^2 - 6x \\ \hline & x - 2 \\ & x - 2 \\ \hline \end{array}$$

$\therefore$
$f(x) = (x - 2)(x^2 + 3x + 1)$

c let $f(x) = 20 + 11x - 8x^2 + x^3$
$f(1) = 24$, $f(2) = 18$, $f(-1) = 0$
$\therefore (x + 1)$ is a factor

$$\begin{array}{r|l} x+1 & x^2 - 9x + 20 \\ \hline & x^3 - 8x^2 + 11x + 20 \\ & x^3 + x^2 \\ \hline & -9x^2 + 11x \\ & -9x^2 - 9x \\ \hline & 20x + 20 \\ & 20x + 20 \\ \hline \end{array}$$

$\therefore$
$f(x) = (x + 1)(x^2 - 9x + 20)$
$= (x + 1)(x - 4)(x - 5)$

d let $f(x) = 3x^3 - 4x^2 - 35x + 12$
$f(1) = -24$, $f(2) = -50$,
$f(-1) = 40$, $f(-2) = 42$
$f(3) = -48$, $f(-3) = 0$
$\therefore (x + 3)$ is a factor

$$\begin{array}{r|l} x+3 & 3x^2 - 13x + 4 \\ \hline & 3x^3 - 4x^2 - 35x + 12 \\ & 3x^3 + 9x^2 \\ \hline & -13x^2 - 35x \\ & -13x^2 - 39x \\ \hline & 4x + 12 \\ & 4x + 12 \\ \hline \end{array}$$

$\therefore$
$f(x) = (x + 3)(3x^2 - 13x + 4)$
$= (x + 3)(3x - 1)(x - 4)$

e let $f(x) = x^3 + 8$
$f(1) = 9$, $f(2) = 16$
$f(-1) = 7$, $f(-2) = 0$
$\therefore (x + 2)$ is a factor

$$\begin{array}{r|l} x+2 & x^2 - 2x + 4 \\ \hline & x^3 + 0x^2 + 0x + 8 \\ & x^3 + 2x^2 \\ \hline & -2x^2 + 0x \\ & -2x^2 - 4x \\ \hline & 4x + 8 \\ & 4x + 8 \\ \hline \end{array}$$

$\therefore$
$f(x) = (x + 2)(x^2 - 2x + 4)$

f let $f(x) = 12 + 29x + 8x^2 - 4x^3$
$f(1) = 45$, $f(2) = 70$,
$f(-1) = -5$, $f(-2) = 18$
$f(3) = 63$, $f(-3) = 105$
$f(4) = 0$
$\therefore (x - 4)$ is a factor

$$\begin{array}{r|l} x-4 & -4x^2 - 8x - 3 \\ \hline & -4x^3 + 8x^2 + 29x + 12 \\ & -4x^3 + 16x^2 \\ \hline & -8x^2 + 29x \\ & -8x^2 + 32x \\ \hline & -3x + 12 \\ & -3x + 12 \\ \hline \end{array}$$

$\therefore$
$f(x) = (x - 4)(-4x^2 - 8x - 3)$
$= -(x - 4)(4x^2 + 8x + 3)$
$= (4 - x)(2x + 1)(2x + 3)$

10 **a** let f(x) = $x^3 - x^2 - 10x - 8$
f(1) = −18, f(2) = −24,
f(−1) = 0
∴ (x + 1) is a factor

$$\begin{array}{r} x^2 - 2x - 8 \\ x+1 \overline{)\, x^3 - x^2 - 10x - 8} \\ \underline{x^3 + x^2} \\ -2x^2 - 10x \\ \underline{-2x^2 - 2x} \\ -8x - 8 \\ \underline{-8x - 8} \end{array}$$

∴
(x + 1)(x^2 − 2x − 8) = 0
(x + 1)(x + 2)(x − 4) = 0
x = −2, −1, 4

b let f(x) = $x^3 + 2x^2 - 9x - 18$
f(1) = −24, f(2) = −20
f(−1) = −8, f(−2) = 0
∴ (x + 2) is a factor

$$\begin{array}{r} x^2 + 0x - 9 \\ x+2 \overline{)\, x^3 + 2x^2 - 9x - 18} \\ \underline{x^3 + 2x^2} \\ 0x^2 - 9x \\ \underline{0x^2 + 0x} \\ -9x - 18 \\ \underline{-9x - 18} \end{array}$$

∴
(x + 2)(x^2 − 9) = 0
(x + 2)(x + 3)(x − 3) = 0
x = −3, −2, 3

c let f(x) = $4x^3 - 12x^2 + 9x - 2$
f(1) = −1, f(2) = 0
∴ (x − 2) is a factor

$$\begin{array}{r} 4x^2 - 4x + 1 \\ x-2 \overline{)\, 4x^3 - 12x^2 + 9x - 2} \\ \underline{4x^3 - 8x^2} \\ -4x^2 + 9x \\ \underline{-4x^2 + 8x} \\ x - 2 \\ \underline{x - 2} \end{array}$$

∴
(x − 2)(4x^2 − 4x + 1) = 0
(x − 2)(2x − 1)^2 = 0
x = $\frac{1}{2}$, 2

d let f(x) = $x^3 - 5x^2 + 3x + 1$
f(1) = 0
∴ (x − 1) is a factor

$$\begin{array}{r} x^2 - 4x - 1 \\ x-1 \overline{)\, x^3 - 5x^2 + 3x + 1} \\ \underline{x^3 - x^2} \\ -4x^2 + 3x \\ \underline{-4x^2 + 4x} \\ -x + 1 \\ \underline{-x + 1} \end{array}$$

∴
(x − 1)(x^2 − 4x − 1) = 0
x = 1 or $\frac{4 \pm \sqrt{16+4}}{2}$
x = 1, 2 ± $\sqrt{5}$

e let f(x) = $x^3 + 4x^2 - 9x - 6$
f(1) = −10, f(2) = 0
∴ (x − 2) is a factor

$$\begin{array}{r} x^2 + 6x + 3 \\ x-2 \overline{)\, x^3 + 4x^2 - 9x - 6} \\ \underline{x^3 - 2x^2} \\ 6x^2 - 9x \\ \underline{6x^2 - 12x} \\ 3x - 6 \\ \underline{3x - 6} \end{array}$$

∴
(x − 2)(x^2 + 6x + 3) = 0
x = 2 or $\frac{-6 \pm \sqrt{36-12}}{2}$
x = 2, −3 ± $\sqrt{6}$

f let f(x) = $x^3 - 14x + 15$
f(1) = 2, f(2) = −5, f(−1) = 28,
f(−2) = 35, f(3) = 0
∴ (x − 3) is a factor

$$\begin{array}{r} x^2 + 3x - 5 \\ x-3 \overline{)\, x^3 + 0x^2 - 14x + 15} \\ \underline{x^3 - 3x^2} \\ 3x^2 - 14x \\ \underline{3x^2 - 9x} \\ -5x + 15 \\ \underline{-5x + 15} \end{array}$$

∴
(x − 3)(x^2 + 3x − 5) = 0
x = 3 or $\frac{-3 \pm \sqrt{9+20}}{2}$
x = 3, $\frac{1}{2}$(−3 ± $\sqrt{29}$)

11 **a** f(2) = 0
∴ 16 − 4 − 30 + c = 0
c = 18

b

$$\begin{array}{r} 2x^2 + 3x - 9 \\ x-2 \overline{)\, 2x^3 - x^2 - 15x + 18} \\ \underline{2x^3 - 4x^2} \\ 3x^2 - 15x \\ \underline{3x^2 - 6x} \\ -9x + 18 \\ \underline{-9x + 18} \end{array}$$

∴ f(x) ≡ (x − 2)(2x^2 + 3x − 9)
≡ (x − 2)(2x − 3)(x + 3)

12 **a** g(−1) = 0
∴ −1 + p + 13 + q = 0
p + q + 12 = 0 (1)
g(3) = 0
∴ 27 + 9p − 39 + q = 0
9p + q − 12 = 0 (2)
(2) − (1) ⇒ 8p − 24 = 0 ⇒ p = 3
sub (1) ⇒ 3 + q + 12 = 0 ⇒ q = −15

b (x + 1)(x − 3)(ax + b) ≡ $x^3 + 3x^2 - 13x - 15$
by inspection
g(x) ≡ (x + 1)(x − 3)(x + 5)
g(x) = 0 ⇒ (x + 1)(x − 3)(x + 5) = 0
x = −5, −1 or 3

13 **a** $= f(2) = 8 + 16 - 2 + 6 = 28$

b $= f(-1) = -1 - 2 - 7 + 1 = -9$

c $= f(-5) = -250 + 25 - 45 + 17 = -163$

d $= f(\frac{1}{2}) = 1 + 1 - 3 - 3 = -4$

e $= f(-\frac{1}{2}) = -\frac{1}{4} - \frac{3}{4} + 10 - 7 = 2$

f $= f(\frac{2}{3}) = \frac{8}{9} - \frac{8}{3} + \frac{4}{3} - 7 = -7\frac{4}{9}$

14 $f(2) = 5$

$\therefore\ 8 - 16 + 10 + c = 5$

$c = 3$

15 $f(\frac{1}{2}) = -2$

$\therefore\ \frac{1}{4} - \frac{9}{4} + \frac{1}{2}k + 5 = -2$

$k = -10$

16 **a** $f(-3) = 22$

$\therefore\ -54 + 9a + 13 = 22$

$a = 7$

b $f(x) = 2x^3 + 7x^2 + 13$

remainder $= f(4)$

$= 128 + 112 + 13$

$= 253$

17 **a** $f(-1) = 0$

$\therefore\ -p + q - q + 3 = 0$

$p = 3$

b $f(x) = 3x^3 + qx^2 + qx + 3$

$f(2) = 15$

$\therefore\ 24 + 4q + 2q + 3 = 15$

$q = -2$

18 **a** $p(3) = 0$

$\therefore\ 27 + 9a + 27 + b = 0$

$9a + b = -54 \quad (1)$

b $p(-2) = -30$

$\therefore\ -8 + 4a - 18 + b = -30$

$4a + b = -4 \quad (2)$

$(1) - (2) \Rightarrow\ 5a = -50$

$\therefore\ a = -10,\ b = 36$

19 $f(-1) = 3$

$\therefore\ -4 - 6 - m + n = 3$

$n - m = 13 \quad (1)$

$f(\frac{1}{2}) = 15$

$\therefore\ \frac{1}{2} - \frac{3}{2} + \frac{1}{2}m + n = 15$

$n + \frac{1}{2}m = 16 \quad (2)$

$(2) - (1) \Rightarrow\ \frac{3}{2}m = 3$

$\therefore\ m = 2,\ n = 15$

20 **a** $g(4) = 39$

$\therefore\ 64 + 4c + 3 = 39$

$c = -7$

b $g(x) = x^3 - 7x + 3$

$$
\begin{array}{r}
x^2 - 2x - 3 \\
x + 2 \overline{)\ x^3 + 0x^2 - 7x + 3} \\
\underline{x^3 + 2x^2} \\
-2x^2 - 7x \\
\underline{-2x^2 - 4x} \\
-3x + 3 \\
\underline{-3x - 6} \\
9
\end{array}
$$

quotient $= x^2 - 2x - 3$

remainder $= 9$

1 **a** $f(-2) = 0 \Rightarrow -8 - 20 - 2a + b = 0$

$\Rightarrow -2a + b = 28 \quad (1)$

$f(3) = 0 \Rightarrow 27 - 45 + 3a + b = 0$

$\Rightarrow 3a + b = 18 \quad (2)$

$(2) - (1) \quad 5a = -10 = 0 \Rightarrow a = -2$

sub. (1) $\Rightarrow b = 24$

b $f(x) \equiv x^3 - 5x^2 - 2x + 24$

$(x + 2)(x - 3)(ax + b) \equiv x^3 - 5x^2 - 2x + 24$

by inspection

$f(x) \equiv (x + 2)(x - 3)(x - 4)$

2 $f(k) = 8f(\frac{1}{2}k)$

$8k^3 - k^2 + 7 = 8(k^3 - \frac{1}{4}k^2 + 7)$

$8k^3 - k^2 + 7 = 8k^3 - 2k^2 + 56$

$k^2 = 49$

$k = \pm 7$

3 **a** $f(2) = 24 - 4 - 24 + 4 = 0$

$\therefore (x - 2)$ is a factor of $f(x)$

b

$$\begin{array}{r} 3x^2 + 5x - 2 \\ x - 2 \overline{)\ 3x^3 - x^2 - 12x + 4} \\ \underline{3x^3 - 6x^2} \\ 5x^2 - 12x \\ \underline{5x^2 - 10x} \\ -2x + 4 \\ \underline{-2x + 4} \end{array}$$

$\therefore f(x) = (x - 2)(3x^2 + 5x - 2)$

$= (x - 2)(3x - 1)(x + 2)$

$f(x) = 0 \Rightarrow (x - 2)(3x - 1)(x + 2) = 0$

$x = -2, \frac{1}{3}$ or 2

4 $6 + 7x - x^3 = 0$

let $f(x) = 6 + 7x - x^3$

$f(1) = 12, \ f(2) = 12, \ f(-1) = 0$

$\therefore (x + 1)$ is a factor of $f(x)$

$$\begin{array}{r} -x^2 + x + 6 \\ x + 1 \overline{)\ -x^3 + 0x^2 + 7x + 6} \\ \underline{-x^3 - x^2} \\ x^2 + 7x \\ \underline{x^2 + x} \\ 6x + 6 \\ \underline{6x + 6} \end{array}$$

$\therefore (x + 1)(-x^2 + x + 6) = 0$

$-(x + 1)(x - 3)(x + 2) = 0$

$x = -2, -1, 3$

$\therefore (-2, 0), (-1, 0)$ and $(3, 0)$

5 **a** $f(-1) = -4$

$\therefore\ -3 + p - 8 + q = -4$

$p + q = 7 \qquad (1)$

$f(2) = 80$

$\therefore\ 24 + 4p + 16 + q = 80$

$4p + q = 40 \qquad (2)$

$(2) - (1) \Rightarrow 3p = 33$

$\therefore\ p = 11,\ q = -4$

b $f(x) \equiv 3x^3 + 11x^2 + 8x - 4$

$f(-2) = -24 + 44 - 16 - 4 = 0$

$\therefore\ (x + 2)$ is a factor

c

$$\begin{array}{r|l} & 3x^2 + 5x - 2 \\ \hline x + 2 & 3x^3 + 11x^2 + 8x - 4 \\ & \underline{3x^3 + 6x^2} \\ & \quad 5x^2 + 8x \\ & \quad \underline{5x^2 + 10x} \\ & \qquad -2x - 4 \\ & \qquad \underline{-2x - 4} \end{array}$$

$\therefore\ f(x) = (x + 2)(3x^2 + 5x - 2)$

$= (3x - 1)(x + 2)^2$

$\therefore\ f(x) = 0 \Rightarrow x = -2 \text{ or } \frac{1}{3}$

7 **a** $= f(-1) = -1 + 7 - 14 + 3 = -5$

b

$$\begin{array}{r|l} & n^2 + 6n + 8 \\ \hline n + 1 & n^3 + 7n^2 + 14n + 3 \\ & \underline{n^3 + n^2} \\ & \quad 6n^2 + 14n \\ & \quad \underline{6n^2 + 6n} \\ & \qquad 8n + 3 \\ & \qquad \underline{8n + 8} \\ & \qquad -5 \end{array}$$

$\therefore\ f(n) = (n + 1)(n^2 + 6n + 8) - 5$

$f(n) = (n + 1)(n + 2)(n + 4) - 5$

c $(n + 1)$ and $(n + 2)$ are consecutive integers

$\therefore$ either $(n + 1)$ or $(n + 2)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4) - 5$ is odd

6 **a** let $f(x) = x^3 - 4x^2 - 7x + 10$

$f(1) = 1 - 4 - 7 + 10 = 0$

$\therefore\ (x - 1)$ is a factor

$$\begin{array}{r|l} & x^2 - 3x - 10 \\ \hline x - 1 & x^3 - 4x^2 - 7x + 10 \\ & \underline{x^3 - x^2} \\ & \quad -3x^2 - 7x \\ & \quad \underline{-3x^2 + 3x} \\ & \qquad -10x + 10 \\ & \qquad \underline{-10x + 10} \end{array}$$

$\therefore\ (x - 1)(x^2 - 3x - 10) = 0$

$(x - 1)(x + 2)(x - 5) = 0$

$x = -2,\ 1,\ 5$

b $y^2 = x$ in part **a**

$y^2 = 1,\ 5 \text{ or } -2$ [no solutions]

$y = \pm 1, \pm\sqrt{5}$

1 **a** $f(-2) = -8 + 4 + 44 - 40 = 0$
$\therefore$ $(x + 2)$ is a factor of $f(x)$

b

$$\begin{array}{r} x^2 - x - 20 \\ x+2 \overline{)\ x^3 + x^2 - 22x - 40} \\ \underline{x^3 + 2x^2} \\ -x^2 - 22x \\ \underline{-x^2 - 2x} \\ -20x - 40 \\ \underline{-20x - 40} \end{array}$$

$\therefore f(x) \equiv (x + 2)(x^2 - x - 20)$
$\equiv (x + 2)(x + 4)(x - 5)$

c $f(x) = 0 \Rightarrow (x + 2)(x + 4)(x - 5) = 0$
$x = -4, -2$ or 5

2 **a** $f(2) = f(-3)$
$\therefore$ $8 - 8 + 2k + 1 = -27 - 18 - 3k + 1$
$k = -9$

b $= f(-2) = -8 - 8 + 18 + 1 = 3$

3 **a** $= p(-2) = -16 - 36 + 4 + 11 = -37$

b

$$\begin{array}{r} 2x^2 - x - 6 \\ x-4 \overline{)\ 2x^3 - 9x^2 - 2x + 11} \\ \underline{2x^3 - 8x^2} \\ -x^2 - 2x \\ \underline{-x^2 + 4x} \\ -6x + 11 \\ \underline{-6x + 24} \\ -13 \end{array}$$

$\therefore$ quotient $= 2x^2 - x - 6$
remainder $= -13$

4 **a** A is $(0, 12)$

b $x = 1$ is a root of $y = 0$
$\therefore$ $(x - 1)$ is a factor of y

$$\begin{array}{r} x^2 - 4x - 12 \\ x-1 \overline{)\ x^3 - 5x^2 - 8x + 12} \\ \underline{x^3 - x^2} \\ -4x^2 - 8x \\ \underline{-4x^2 + 4x} \\ -12x + 12 \\ \underline{-12x + 12} \end{array}$$

$\therefore y = (x - 1)(x^2 - 4x - 12)$
$= (x - 1)(x + 2)(x - 6)$
$\therefore y = 0$ when $x = -2, 1$ or 6
$\therefore$ B is $(-2, 0)$ and D is $(6, 0)$

5 **a** $f(1) = 0$
$\therefore$ $1 - 3 + k + 8 = 0$
$k = -6$

b

$$\begin{array}{r} x^2 - 2x - 8 \\ x-1 \overline{)\ x^3 - 3x^2 - 6x + 8} \\ \underline{x^3 - x^2} \\ -2x^2 - 6x \\ \underline{-2x^2 + 2x} \\ -8x + 8 \\ \underline{-8x + 8} \end{array}$$

$\therefore f(x) = (x - 1)(x^2 - 2x - 8)$
$= (x - 1)(x + 2)(x - 4)$
$f(x) = 0 \Rightarrow x = -2, 1, 4$

6 let $f(x) = 2x^3 + x^2 - 13x + 6$
$f(1) = -4$, $f(2) = 0$
$\therefore$ $(x - 2)$ is a factor of $f(x)$

$$\begin{array}{r} 2x^2 + 5x - 3 \\ x-2 \overline{)\ 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 - 4x^2} \\ 5x^2 - 13x \\ \underline{5x^2 - 10x} \\ -3x + 6 \\ \underline{-3x + 6} \end{array}$$

$\therefore$ $(x - 2)(2x^2 + 5x - 3) = 0$
$(x - 2)(2x - 1)(x + 3) = 0$
$x = -3, \frac{1}{2}, 2$

7 **a** $p(-1) = 3$

$\therefore\ -b + a + 10 + b = 3$

$a = -7$

b $p(\frac{1}{3}) = -1$

$\therefore\ \frac{1}{27}b - \frac{7}{9} - \frac{10}{3} + b = -1$

$b - 21 - 90 + 27b = -27$

$b = 3$

8 **a** $= f(-1) = -1 - 7 - 1 + 10 = 1$

b $x^3 - 7x^2 + x + 10 = 1$

$x^3 - 7x^2 + x + 9 = 0$

$x = -1$ is solution $\therefore\ (x + 1)$ is factor

$$\begin{array}{r} x^2 - 8x + 9 \\ x+1 \overline{)\ x^3 - 7x^2 + x + 9} \\ x^3 + x^2 \\ \hline -8x^2 + x \\ -8x^2 - 8x \\ \hline 9x + 9 \\ 9x + 9 \\ \hline \end{array}$$

$\therefore\ (x + 1)(x^2 - 8x + 9) = 0$

$x = -1,\ \frac{8 \pm \sqrt{64 - 36}}{2} = -1,\ 4 \pm \sqrt{7}$

9 $f(\frac{2}{3}) = 6$

$\therefore\ \frac{8}{9} + \frac{4}{9}k - \frac{14}{3} + 2k = 6$

$8 + 4k - 42 + 18k = 54$

$22k = 88$

$k = 4$

10 **a** $f(3) = 54 - 63 + 12 - 3 = 0$

$\therefore\ (x - 3)$ is a factor of $f(x)$

b

$$\begin{array}{r} 2x^2 - x + 1 \\ x-3 \overline{)\ 2x^3 - 7x^2 + 4x - 3} \\ 2x^3 - 6x^2 \\ \hline -x^2 + 4x \\ -x^2 + 3x \\ \hline x - 3 \\ x - 3 \\ \hline \end{array}$$

$\therefore\ f(x) \equiv (x - 3)(2x^2 - x + 1)$

c $f(x) = 0 \Rightarrow (x - 3)(2x^2 - x + 1) = 0$

$x = 3$ or $2x^2 - x + 1 = 0$

for $2x^2 - x + 1 = 0,\ b^2 - 4ac = -7$

$b^2 - 4ac < 0 \quad \Rightarrow$ no real roots

$\therefore$ only one real solution

11 **a** $f(2) = 0$

$\therefore\ 8 + 2p + q = 0$

$q = -2p - 8$

b $f(-1) = -15$

$\therefore\ -1 - p + q = -15$

$q = p - 14$

$\therefore\ p - 14 = -2p - 8$

$p = 2,\ q = -12$

12 $f(-3) = 0\ \therefore\ (x + 3)$ is a factor of $f(x)$

$$\begin{array}{r} x^2 + x - 3 \\ x+3 \overline{)\ x^3 + 4x^2 + 0x - 9} \\ x^3 + 3x^2 \\ \hline x^2 + 0x \\ x^2 + 3x \\ \hline -3x - 9 \\ -3x - 9 \\ \hline \end{array}$$

$\therefore\ f(x) = (x + 3)(x^2 + x - 3)$

other solutions given by $x^2 + x - 3 = 0$

$x = \frac{-1 \pm \sqrt{1 + 12}}{2} = \frac{-1 \pm \sqrt{13}}{2}$

$x = -2.30$ or 1.30

13 **a** $f(-2) = -7$

$\therefore\ (-2 + k)^3 - 8 = -7$

$(k - 2)^3 = 1$

$k = 3$

b $f(x) \equiv (x + 3)^3 - 8$

$\therefore\ f(-1) = 2^3 - 8 = 0$

$\therefore\ (x + 1)$ is a factor

14 **a** $= f(-2) = -8 - 16 + 14 + 8 = -2$

b $c = 2$

c $g(x) \equiv x^3 - 4x^2 - 7x + 10$

$$\begin{array}{r} x^2 - 6x + 5 \\ x + 2 \overline{)\ x^3 - 4x^2 - 7x + 10} \\ \underline{x^3 + 2x^2} \\ -6x^2 - 7x \\ \underline{-6x^2 - 12x} \\ 5x + 10 \\ \underline{5x + 10} \end{array}$$

$\therefore\ g(x) = (x + 2)(x^2 - 6x + 5)$

$= (x + 2)(x - 1)(x - 5)$

$g(x) = 0 \ \Rightarrow\ x = -2, 1, 5$

15 **a** $f(\frac{1}{2}k) = 4$

$\therefore\ \frac{1}{8}k^3 - 2k + 1 = 4$

$k^3 - 16k + 8 = 32$

$k^3 - 16k - 24 = 0 \qquad (1)$

b $f(-k) = 1$

$\therefore\ -k^3 + 4k + 1 = 1$

$k^3 = 4k$

sub (1) $\Rightarrow\ 4k - 16k - 24 = 0$

$12k = -24$

$k = -2$